Chapter 6

A Hybrid DEA-PROMETHEE II Method: A Complete Ranking of DMUs

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ABSTRACT

Efficiency evaluation of organisation is an important aspect of strategic management. The top management must know the level of their organisation's efficiency to figure out necessary changes and adjustments, if any. This is to ensure the organisation remains competitive. Data Envelopment Analysis (DEA) is a methodology that has been developed to measure the efficiency of a set decision making units (DMUs). The most significant advantage of DEA approach is its ability to handle multiple inputs and outputs. Nevertheless, classical DEA has some drawbacks. One drawback of DEA is poor discrimination power in which DEA produces a solution that recognizes many DMUs as efficient units. Ranking DMUs and choosing the most efficient unit is important for decision makers. However, classical DEA can only classify DMUs as efficient and inefficient units. Since all efficient units have the same efficiency score of unity, it is not possible to do a complete ranking among these units. Thus, this study integrates PROMETHEE II into classical DEA to rank the DMUs completely. PROMETHEE II is a Multi-Attribute Decision Making method that is based on a mutual comparison of each alternative pair with respect to each of the selected criteria allowing the rankings of the alternatives from the best to the worst. PROMETHEE II is suitable for this study since this method can provide the complete ranking of the DMUs. The objectives of this study are to measure efficiency and provide a complete ranking of DMUs under study. Unlike the previous studies that integrated DEA and PROMETHEE II to reduce the number of efficient DMUs, the proposed method for the current study retains the number of efficient units and yet can fully rank the efficient units. This is the novelty of this study. This information helps these organisations in managing their resources more efficiently. The efficient DMUs can be regarded as the model for the
other units for benchmarking purposes. The inefficient DMUs can emulate the best practice of efficient DMUs to attain a higher level of efficiency.

**Key Words**: DEA, PROMETHEE II, ranking, efficiency & Decision Making Units.

### 1. INTRODUCTION

In today’s competitive world, efficiency measurements of an organisation have become increasingly important. However, measuring efficiency is an enormous challenge because most organisations utilize multiple inputs to produce multiple outputs. Data Envelopment Analysis (DEA) is a nonparametric method that has become a powerful approach to measure efficiency of a set of comparable entities known as decision making units (DMUs). It has been widely applied in various sectors, such as the insurance, health, education and other sectors. One of the strengths of DEA is it can handle multiple inputs and outputs. However, DEA has some drawbacks. One drawback of standard DEA is lack of discrimination power where it identifies many DMUs as efficient units (Bal, Örkcü, & Çelebioğlu, 2010). The standard DEA can only classify DMUs as efficient or inefficient units. It cannot discriminate against efficient DMUs. Therefore, complete ranking among the efficient units is not possible even though ranking DMUs is important for decision makers.

PROMETHEE method; the preference ranking organisation method for enrichment evaluations, has been developed to analyse an evaluation problem. In this study, a hybrid DEA and PROMETHEE II method is presented for ranking efficient DMUs, hence provides a complete ranking of the DMUs. PROMETHEE II is chosen over other versions of PROMETHEE because of its ability to give a complete ranking of alternatives from the best to the worst one. Bagherikahvarin and De Smet (2016) have recommended the use of DEA and PROMETHEE II method to increase the discrimination power in DEA by reducing the number of efficient DMUs. Our proposed method is to retain the number of efficient units, nonetheless it is able to fully rank the efficient units. This is the novelty of this study. We illustrate the application of this method to rank life insurance and takaful companies in Malaysia.

### 2. LITERATURE REVIEW

Numerous approaches have been proposed to rank efficient units in DEA. Because traditional DEA failed to identify the most efficient DMU, many researchers have integrated DEA with other methods to rank efficient units in DEA, particularly to increase the discrimination power of DEA. These include super efficiency models, Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), PROMETHEE and PROMETHEE II. The method of combining DEA with super efficiency models was introduced by Andersen and Petersen (1993). The work of
Anderson and Peterson was later extended to resolve other crucial issues such as outlier detection, sensitivity analysis, infeasibility and scale classification.

Besides super-efficiency, another ranking method to rank efficient DMUs that has been applied in past studies is DEA-TOPSIS. Chitnis and Vaidya (2016) have computed and ranked performance efficiency of various branches of an Indian bank by using DEA and TOPSIS method to overcome the difficulty of assigning unique ranking. Previously, Lotfi, Fallahnejad and Navidi (2011) have used DEA-TOPSIS to rank 20 Iranian bank branches. Later, Mandić, Delibašić, Knežević and Benković (2017) have utilised Fuzzy Analytic Hierarchy Process (FAHP) and TOPSIS. The study was carried out to assess Serbian insurance companies’ efficiency in the period from 2007 to 2014. Other researchers that have also employed Fuzzy AHP and TOPSIS are Akkoc and Vatansever (2013) and Amile, Sedaghat and Poorhossein (2013). The latter have applied this method to conduct performance evaluation of state-owned, fully and partially private banks in Iran. In addition to super-efficiency and TOPSIS integrated models, past studies have also employed PROMETHEE in an attempt to measure the performance of the financial sector.

PROMETHEE method was first developed by Jean-Pierre Brans in 1982 and there are several versions of the PROMETHEE methods including PROMETHEE I and PROMETHEE II. PROMETHEE I is for partial ranking while the PROMETHEE II for complete ranking of the alternatives. Uzar (2013) has measured the financial performance of public banks in Turkey by applying the PROMETHEE method to compare the financial performance of public banks for two (2) periods: pre-crisis (2002-2007) and post crisis (2008-2012) period. Gökalp (2015) then has extended the research of Turkish banks in different periods: pre crisis (2006-2008) and post crisis (2009-2012) by applying PROMETHEE II.

Based on the previous literature, although there has been an increasing interest in assigning ranking performance of DMUs, the method of DEA and PROMETHEE II has not been much adopted, particularly for insurance sectors

3. METHODOLOGY

3.1. Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a non-parametric method that was developed by Farrell in 1957 (Färe, Grosskopf & Norris, 1994) and later extended by Charnes et al. (1978). For this paper, we propose an input-oriented CCR model. The mathematical formulation is in the following form:

Supposedly there are n DMUs where each $DMU_o (o = 1,2,3,...,n)$ utilizes $m$ inputs and $s$ outputs.
Maximize \( \theta_o = \sum_{r=1}^{s} u_r y_{ro} \)

subject to

\[
\sum_{r=1}^{m} v_i x_{io} = 1, \\
\sum_{r=1}^{s} u_r y_{ij} - \sum_{i=1}^{m} v_i x_{ij} \leq 0; \quad j = 1, 2, \ldots, n \\
u_r, v_i \geq 0.
\]

If \( \theta_o = 1 \), it means that \( DMU_o \) is efficient relative to other units, otherwise it is inefficient.

### 3.2. Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE II)

The PROMETHEE II outranking method is used to generate a complete ranking of DMUs by making pairwise comparisons of all the DMUs under study. There are seven (7) steps in the framework of PROMETHEE II (Brans, Vincke, & Mareschal, 1986; Sen, Datta, Patel, & Mahapatra, 2015; Athawale & Chakraborty, 2011) to provide the complete ranking of the DMUs as follows:

Step 1: Construct the decision matrix.

Step 2: Normalize the decision matrix by using equation (2) and (3) for beneficial criteria and non-beneficial criteria, respectively.

\[
R_{ij} = \left[ \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})} \right] \quad \text{for } i = 1, 2, 3, \ldots, m; \quad j = 1, 2, 3, \ldots, n.
\]

(2)

\[
R_{ij} = \left[ \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})} \right] \quad \text{for } i = 1, 2, 3, \ldots, m; \quad j = 1, 2, 3, \ldots, n.
\]

(3)

Step 3: Calculate the evaluative differences of \( i^{th} \) alternative with respect to another alternative, \( d_j(a, b) \) by using

\[
d_j(a, b) = g_j(a) - g_j(b)
\]

(4)

Step 4: Calculate the preference function, \( P_j(a, b) \) using

\[
P_j(a, b) =\begin{cases} 0 & \text{if } R_{aj} \leq R_{bj} \text{ such that } D(M_a - M_b) \leq 0 \\ R_{aj} - R_{bj} & \text{if } R_{aj} > R_{bj} \text{ such that } D(M_a - M_b) > 0 \end{cases}
\]

(5)

Step 5: Calculate the aggregated preference, \( \pi(a, b) \) by using
\[
\pi(a, b) = \frac{\sum_{j=1}^{n} w_j P_j(a, b)}{\sum_{j=1}^{n} w_j} \quad \text{where} \quad \sum_{j=1}^{n} w_j = 1.
\] (6)

Given that \(\sum_{j=1}^{n} w_j\) is the sum of the weight for criteria.

Step 6: Determine the leaving and the entering outranking flow using equation (7) and (8) respectively.

Leaving (positive) flow for \(a^{th}\) alternative, \(\varphi^+(a)\)

\[
= \frac{1}{m-1} \sum_{b=1}^{m} \pi(b, a) \quad \text{where} \quad (a \neq b)
\] (7)

Entering (negative) flow for \(a^{th}\) alternative, \(\varphi^-(a)\)

\[
= \frac{1}{m-1} \sum_{b=1}^{m} \pi(b, a) \quad \text{where} \quad (a \neq b)
\] (8)

Step 7: Calculate the net outranking flow for each alternative using

\[
\varphi(a) = \varphi^+(a) - \varphi^-(a)
\] (9)

A real-life numerical example about the life insurance and takaful companies in Malaysia is used in the application of the hybrid DEA and PROMETHEE II method to measure efficiency and provide complete ranking of DMUs under study.

4. RESULTS AND DISCUSSION

Table 1 summarizes the efficiency scores of each 22 DMUs which are derived from the DEA model. Since the classical DEA model is formulated based on input and output data of DMUs, the higher the efficiency scores indicate that the DMUs are efficient in utilising their resources to produce an optimal output. However, seven (7) out of 22 DMUs have been considered as the most efficient DMUs during the observed period. This problem occurs because the number of DMUs is not large enough in comparison with the total number of inputs and outputs, which leads to many DMUs being identified as efficient. Thus, the discrimination power of DEA is decreased and this shows that DEA alone is not a good discriminator among DMUs since it cannot give distinctive ranking to fully evaluate the individual efficiency of the seven (7) DMUs mentioned above.

For this project, an integrated DEA model with PROMETHEE II method is employed. For this purpose, we use the net flow scores based on PROMETHEE II to improvise the results from DEA. Findings reveal that by employing PROMETHEE II, the
Table 1: Comparison of result between classical DEA and proposed hybrid DEA PROMETHEE II

<table>
<thead>
<tr>
<th>Decision Making Unit (DMU)</th>
<th>Efficiency Scores</th>
<th>Ranking using classical DEA</th>
<th>Ranking using DEA-PROMETHEE II</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU2</td>
<td>100</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DMU4</td>
<td>100</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>DMU5</td>
<td>100</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>DMU10</td>
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<td>1</td>
<td>4</td>
</tr>
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<td>DMU12</td>
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<td>1</td>
<td>5</td>
</tr>
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<td>DMU14</td>
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<td>DMU19</td>
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<td>DMU9</td>
<td>83.08</td>
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<td>DMU13</td>
<td>74.40</td>
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<td>9</td>
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<td>DMU11</td>
<td>73.83</td>
<td>10</td>
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<td>72.77</td>
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<td>11</td>
</tr>
<tr>
<td>DMU6</td>
<td>72.14</td>
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<td>12</td>
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<td>DMU7</td>
<td>71.60</td>
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<td>13</td>
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<td>68.8</td>
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<td>DMU21</td>
<td>35.56</td>
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</tbody>
</table>

5. CONCLUSION & RECOMMENDATION

One shortfall of DEA is that it has poor discrimination power. Oftentimes, it only identifies DMUs as efficient units and is not able to discriminate efficient units any further. In short, this DEA is unable to provide complete ranking for the DMUs. Hence, this paper proposes the use of hybrid DEA-PROMETHEE II method to fully rank DMUs. It is found that this method can retain the number of efficient DMUs and discriminate further among the efficient DMUs. This information helps top management to seek strategies to improve performance of the organisation, plus the efficient DMUs can be emulated by other inefficient DMUs.

REFERENCES


