Numerical Solution of Estimating Time of Death by Newton's Law of Cooling

Rahmah Shahril, Maznah Banu Mohamed Habiboo Raman & Rasidah Buang

Seremban Campus, Negeri Sembilan Branch,
Universiti Teknologi MARA

rahma949@uitm.edu.my

ABSTRACT

In forensic science, one of the most important issues was to estimate the time of death. Newton's Law of Cooling model has been widely used to estimate the time of death. Numerical method is a complete and specific set of procedures that was used to solve Newton's Law of Cooling model. Two different murder cases were considered in predicting the time of death, and the method of solving this problem are Euler method and Runge-Kutta 4^{th} Order. The range for the solution of estimating time of death for Case 1 was between 9.40 pm and 9.45 pm, while for Case 2 the solution range were from 10.00 pm to 10.05 pm. Based on the relative error, a best method was chosen.

Key Words: Newton's Law of Cooling, Runge-Kutta, Euler

1. INTRODUCTION

Newton's law of cooling defines a relation between a body's cooling level, and the difference between the temperature and the environment's temperature. Since Newton Law of Cooling is a first order ordinary differential equation (ODE), it can be solved by analytical method and numerical method. Numerical method was a technique used for numerical approximation to ODE solutions, also known as 'numerical integration'. Numerical methods for first order ODE are divided into two major categories, linear multistep methods and Runge-Kutta method.

Rodrigo (2015) used Laplace Transform Approach in solving Newton Law of Cooling, and estimated the time of death from temperature by measurements. Their model included an arbitrary, time-dependent function due to the body's thermal conductivity, which is associated with the heat energy in the compartment. While Boahene (2015) performed analytic method which is variable separation technique in solving Newton's Law of Cooling. Boahene said the model assumed a constant ambient temperature, and does not include many parameters in its calculation, which make it largely inaccurate to estimate TOD. The main objective of this paper was to solve Newton's Law of Cooling model analytically and numerically, as well as estimating the time of death using Euler method and Runge-Kutta 4^{th} Order. There are a few limitation, such as secondary data obtained from previous study.

2. LITERATURE REVIEW

One of the significant thing in forensic science before undergoing a thorough inquiry was to determine the time of death (Abdullah et al., 2014). Time passed after death remained a biggest issue for the forensic pathologist, whose dedication plays a major role in medical cases, as forensic experts often have to answer death questions in court (Asante, 2013). Newton's Cooling Law was generally used to estimate the time of death (TOD). According to Newton's Law of Cooling, the rate of heat transferred from the body's surface to the surrounding fluid, that is proportional to the temperature difference between the ground and the fluid (Rodrigo, 2015).

This model is given by,

$$\frac{dT}{dt} = k(T_t - T_s), \quad t_0 \le t \le t_f \tag{1}$$

with initial condition T(0) = T
where,

T_t = temperature at time t , T_s = temperature of the surrounding and k = positive constant

To derive the Euler and Runge-Kutta 4th order method, we divide the interval $[t_0, t_f]$ into N equal subintervals of mesh length h and the mesh points given by $t_n = t_0 + nh$, $n = 1,2,3,..,k.$

The first method used to solve was Euler Method which is express as:

$$T_{n+1} = T_n + hf(t_n, T_n) \tag{2}$$

Euler method is a mathematical solution for differential equation with a given initial value problem. The idea behind the Euler method was to combine several small line segments using the concept of local linearity to estimate the real curve (Biswas, Chatterjee, Mukherjee, & Pal, 2013). The Euler method is both one and multi-step, together with the requirements for stability (Ochoche, 2008). The second method was Runge-Kutta 4th order, which was a common method in numerical analysis to approximate solutions of ordinary differential equations (Hairer & Wanner, 2015) and the equation was:

$$T_{n+1} = T_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{3}$$

$$k_1 = hf(t_n, T_n)$$

$$k_2 = hf(t_n + \frac{h}{2}, T_n + \frac{k_1}{2})$$

$$k_3 = hf(t_n + \frac{h}{2}, T_n + \frac{k_2}{2})$$

$$k_4 = hf(t_n + h, T_n + k_3)$$

Research done by Kwach et al. (2013) found that the differential equation solution will be a function that tracks the complete temperature record over time, allowing them to solve the TOD. In constructing the equation, they stated that if the body is warmer than the ambient temperature, the body will cool down and the derivative will be negative.

3. METHODOLOGY

Case 1: The victim was found by police personnel at 10.40 pm and the body temperature at that exact time was 94.4°F. The body was located in a room where the room temperature is kept constant at 70°F. Not long after the death, the body temperature was decreasing, and assumed that the victim's temperature was 98.6°F at the time of death (Kwach et al., 2013). After 90 minutes, the body temperature was recorded at 89°F.

The exact solution for Case 1 (Kwach et al., 2013) was found to be

$$T = T_s + (T_0 - T_s)e^{-0.00277938t} \tag{4}$$

Based on the problem given, the Newton Law of Cooling can be modelled as $\frac{dT}{dt} = -0.00277938(T - 70)$ (5)

with the initial condition $T(0) = 98.6$. To solve this problem, we let

$$\frac{dT}{dt} = f(t, T) \tag{6}$$

Therefore, the general formula for Euler approximation was

$T_{n+1} = T_n + h(-0.00277938)(T_n - 70)$ for n = 0,1,2,3… with the step size h = 0.1 (7)

For Runge-Kutta 4th order method, the general formula was given by equation (3).

Case 2: A man was killed in his apartment and the body was discovered at 7.00 am. The coroner noted that the room temperature was kept the same level all the time at 70°F. The temperature of the body was 72.5°F the moment it was found, and after 60 minutes, the temperature dropped to 72°F. Let T(*t*) be the temperature *t* in minutes of the body and normal body temperature is 98.6°F (Woodson et al., 1996).

The exact solution for Case 1 (Woodson et al., 1996) is

$$T = T_s + (T_0 - T_s)e^{-0.003719059189t} \tag{8}$$

The Newton Law of Cooling for Case 2 can be modelled as

$$\frac{dT}{dt} = -0.0037190591\,89(T - 70) \tag{9}$$

with the initial condition T(0) = 98.6.

For Runge-Kutta 4th order method, the general numerical solution was given by equation (3), where it needs four values of function in each step of iteration.

4. RESULTS & DISCUSSION

Case 1:

The officer found the body at 10.40 pm and the temperature of the corpse recorded was 94.4°F. After 90 minutes, the temperature was recorded again that showed 89°F. The actual time of death was at 9.42:50 pm, which was 57.16 minutes before it was discovered. Under the same circumstances, Euler Method was used to find the time of death of the victim showed that it had been dead for 57.10 minutes. It could be deduced that the victim died at 9.42:52 pm, that was 57.14 minutes prior to the discovery. Then, Runge-Kutta 4th Order computed that the victim had been dead for 57.15 minutes, where it can be assumed that the victim died at 9.42:51 pm. These results are presented in figure 1.

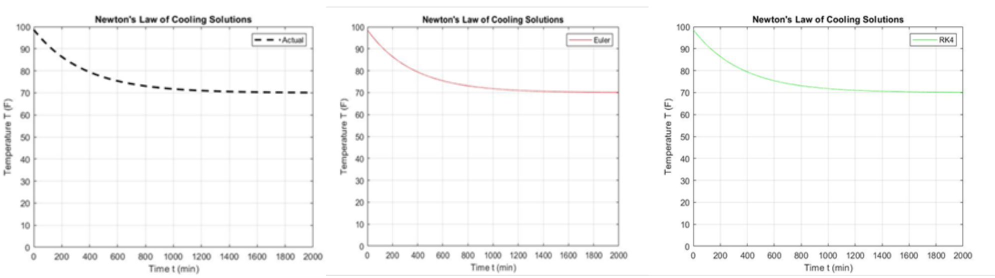

Figure 1: Results for Case 1 in Temperature T(t) using Euler and R-K 4

Case 2:

The body was discovered at 7.00 am. At that time, the temperature was 72.5°F and an hour later, the temperature dropped to 72°F. Table 4.1 showed that the actual time of death was at 10.04:41 pm the night before it was discovered. The body died 10.922 hours before it was retrieved. Euler method was used to determine the time of death too. For this method, it was perceived that the victim died approximately 10.920 hours earlier from the time it was found. The victim died at 10.04:49 pm. The computational using Runge-Kutta 4th order presented that the time of death was at 10.04:42 pm, and that indicated the victim had been dead for 10.922 hours.

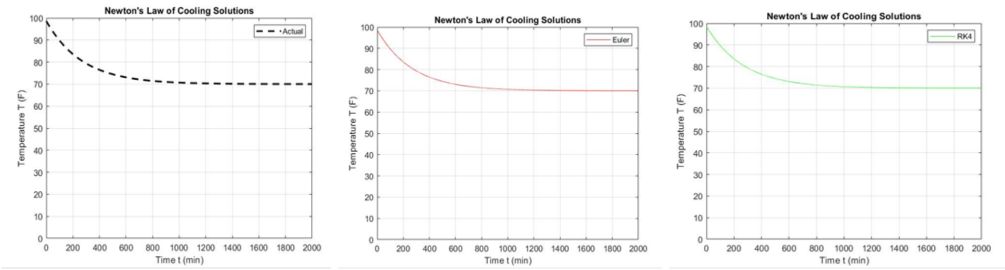

Figure 2: Results for Case 1 in Temperature T(t) using Euler and R-K 4

From the duration obtained, it can be deducted that the victim has been dead for 55 minutes to an hour before it was discovered for Case 1. As for Case 2, it has been identified that the body was not found until almost 11 hours later. Overall, the time of death for Case 1 could be reduced to between 9.40 pm to 9.45 pm, while for Case 2 it was in the interval of 10.00 pm until 10.05 pm. For better estimation time of death, relative error was used to compare the accuracy as summarized in Table 1.

Table 1: Relative Error for Both Cases

Method	Case 1	Case 2
Euler	3.4990×10^{-4}	1.9838×10^{-4}
Runge-Kutta 4th Order	1.7495×10^{-4}	1.5260×10^{-5}

5. CONCLUSION & RECOMMENDATION

Runge-Kutta 4th Order had better estimation time of death as it gave the smallest relative error compared to Euler Method. Other numerical methods, in particular Runge-Kutta Fehlberg Method and Finite Diference Method could be used to estimate the time of death.

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